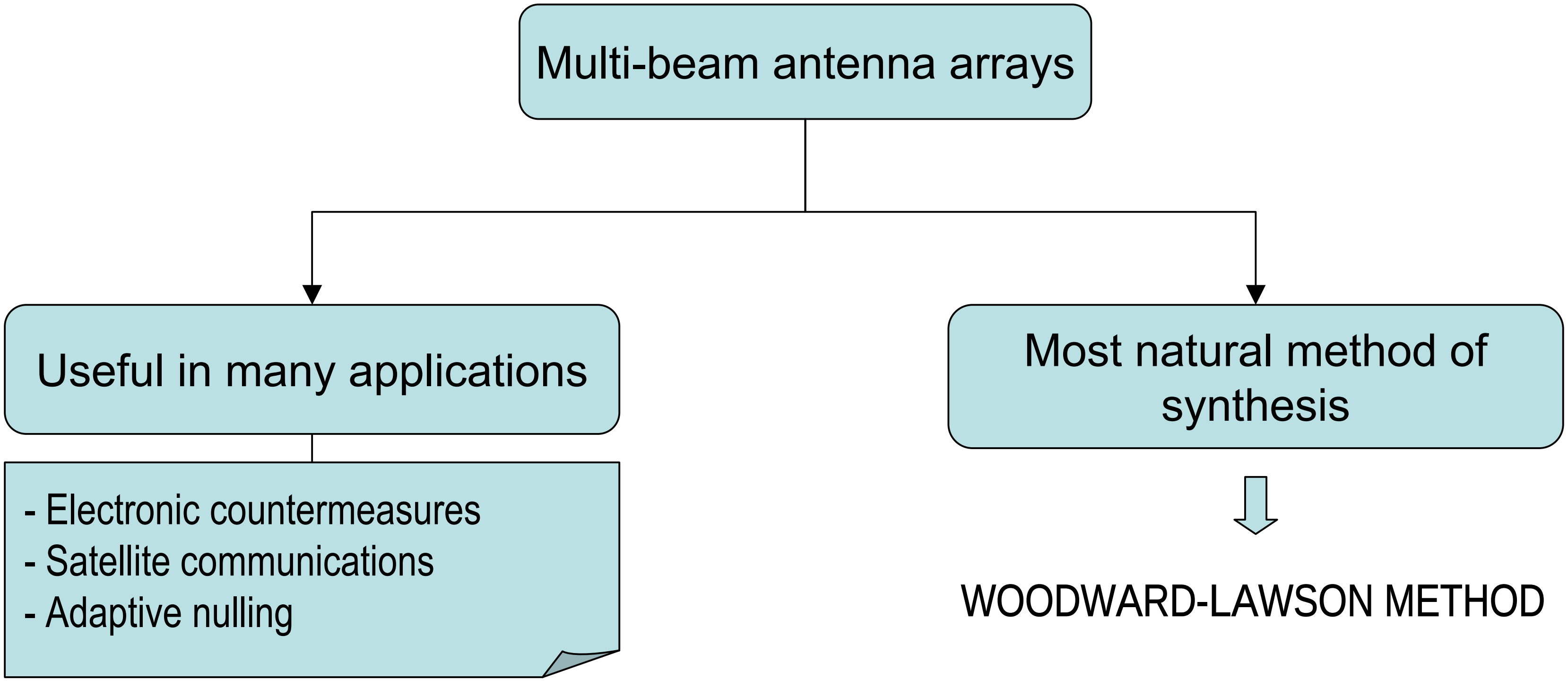
# A simple way of obtaining optimized patterns using the Woodward-Lawson method

J. Fondevila-Gómez, J. A. Rodriguez, F. Ares, and E. Moreno

Dept. of Applied Physics, University of Santiago de Compostela,
15782 Santiago de Compostela (Spain).

# INTRODUCTION



# WOODWARD-LAWSON METHOD

## Advantages

- It is an **analytical**, simple and elegant synthesis method.
- It can be **implemented** in real life using lossless orthogonal beam-forming networks such as the Butler matrix (for example: Rotman lens, a broadband device).

## Limitations

- **Poor control** over the amplitude of the ripple in the shaped region and the heights of side lobes in the unshaped region.
- **Pattern zeros** occur in pairs: There are only half as many adjustments available as there could be for a given number of elements in an array → In the shaped pattern region the number of ripples is half what it could be.

*Causes* ⇓

- It has **zero tolerance** of deviation from the desired field at the sampling points.
- It constrains all the beams to have the same phase → **real field pattern.**

HOW TO IMPROVE THE METHOD?

Previous literature:

*Cid et al., "Shaped power patterns produced by equispaced linear arrays: optimized synthesis using orthogonal sin(Nx)/sin(x) beams", J. Electromagn. Waves Appl, 31, 985-992, 1999*.

- Improved the obtained patterns by **perturbing** the amplitudes and/or phases of the beams.
- This perturbation is performed using the **simulated annealing** technique, which may require a lot of iterations for large arrays.

In this work:

*The beam coefficients are obtained by sampling a previously synthesized pattern using the **Orchard-Elliott method**.*

- The patterns are the same as the obtained by the Orchard-Elliott method.
- The Orchard-Elliott technique is computer-efficient, inexpensive and rapidly convergent.

# DESCRIPTION OF THE METHOD

● **Original Woodward-Lawson method** (*N*-element linear array, interelement spac. *d*)

- **Sampling** the desired field pattern F(θ) in 2M+1 directions located at $\theta_i = \cos^{-1}(i\lambda/Nd)$

  θ → angle measured from enfire, $M=Nd/\lambda$ (chosen to cover the visible region)

- **Approximating** the array factor by:

$$WL(\theta) = \sum_{i=-M}^{M} F(\theta_i) \frac{\sin(Nu_i)}{N\sin(u_i)}, \text{ where } u_i = \frac{\pi d}{\lambda}\left[\cos(\theta) - \cos(\theta_i)\right]$$

  - The coefficient of the *i*-th quasi-sinc beam is exactly $F(\theta_i)$, being all **in phase**.

  - All the component quasi-sinc beams except the i-th are zero at $\theta_i \rightarrow WL(\theta_i)=F(\theta_i)$.

► Although the original Woodward-Lawson method assumes that all the quasi-sinc beams are in phase, we found that **this phase information is critical to obtain good quality patterns**.

  - It is possible to double the number of ripples in shaped region just by introducing a phase in the Woodward-Lawson coefficients → This often improves the ripple level of the power pattern.

- The Woodward-Lawson method always provides a **multiplicity of solutions for the current distributions**.

Excitations obtained by the Woodward-Lawson method $\{I_n\}$

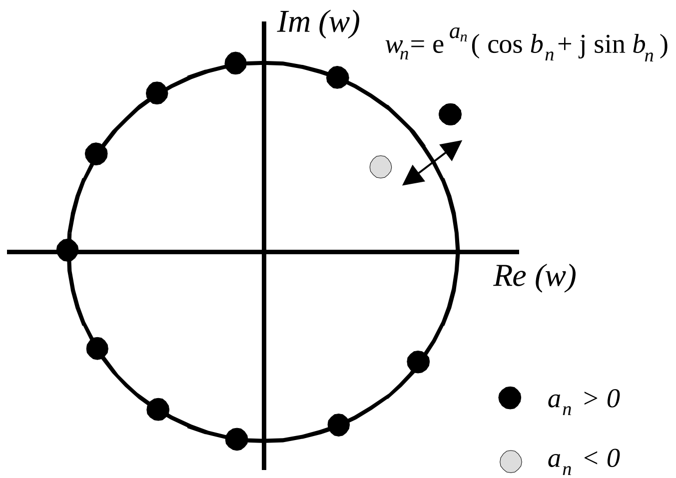
→ Calculation of the roots of the polynomial associated to the array factor: $\{w_n=\exp(a_n+jb_n)\}$

→ Normalized power pattern is not modified when $a_n \rightarrow -a_n$

→ This is equivalent to move the filled roots inside or outside the Schelkunoff unit circle

⇒ Q roots off the unit circle → $2^Q$ currents distributions producing the same normalized power pattern

→ This would allow to alleviate mutual coupling problems




Roots: $(+a_n, b_n)$ or $(-a_n, b_n)$

This multiplicity of solutions exist even if the roots are arranged in pairs (as usual in the original Woodward-Lawson method). In this case there are $3^{Q/2}$ different solutions.

## • The proposed method:

- In this work we obtain the Woodward-Lawson beam coefficients by sampling (keeping both the amplitude and phase) of a previously synthesised pattern.

→ This pattern is obtained using the Orchard-Elliott method.

**Objective pattern** with a desired ripple and sidelobe topography

↓

**Orchard-Elliott method** is used to synthesize this pattern (it allows an excellent control both in the shaped and the sidelobe region)

↓

The amplitude pattern and phase pattern is **sampled** at the $\theta_i$ locations.

↓

Application of the **Woodward-Lawson method** with these samples provides the same pattern as the previously synthesized.

**-** The advantages of this method are not only the quality of the resulting patterns, but also the multiplicity of the provided solutions.

→ This method provides a **multiplicity of solutions for the Woodward-Lawson beam coefficients**.

If the pattern has **Q filled nulls**, the Orchard-Elliott method produces $2^Q$ independent set of excitations that generate the same (normalized) amplitude pattern but different phase patterns

⇓

We also have $2^Q$ **set of samples** to be used for the Woodward-Lawson method.

⇓

Among these sets, we can choose that one that leads to an **easier physical** realization of the array by using, for example, a Rotman lens-fed array.

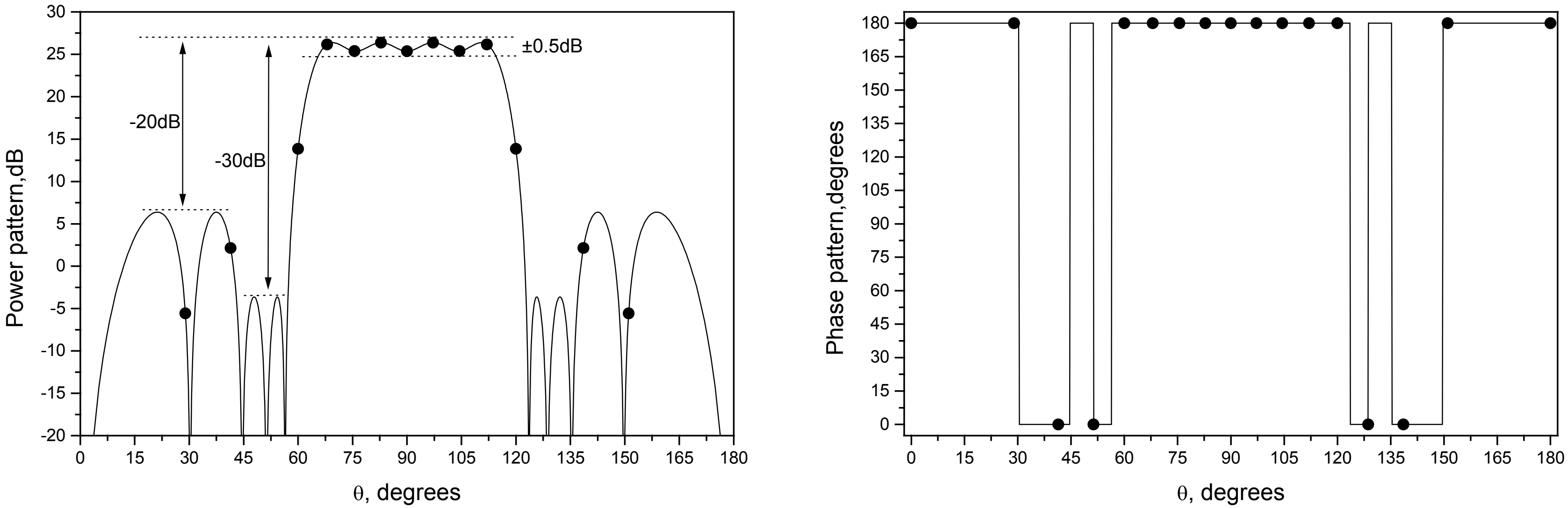
# EXAMPLES

- In all the examples we have used a 16-element ($\lambda$/2)-spaced linear array

**a) Real pattern:**

• Desired pattern: flat-topped beam pattern with a ripple requirement of ±0.5 dB, two -30 dB inner side lobes, and all other sidelobes at -20 dB.

• This pattern is easily synthesized by using an extension of the Orchard-Elliott method (that arranges the filled roots in pairs in order to obtain real patterns).

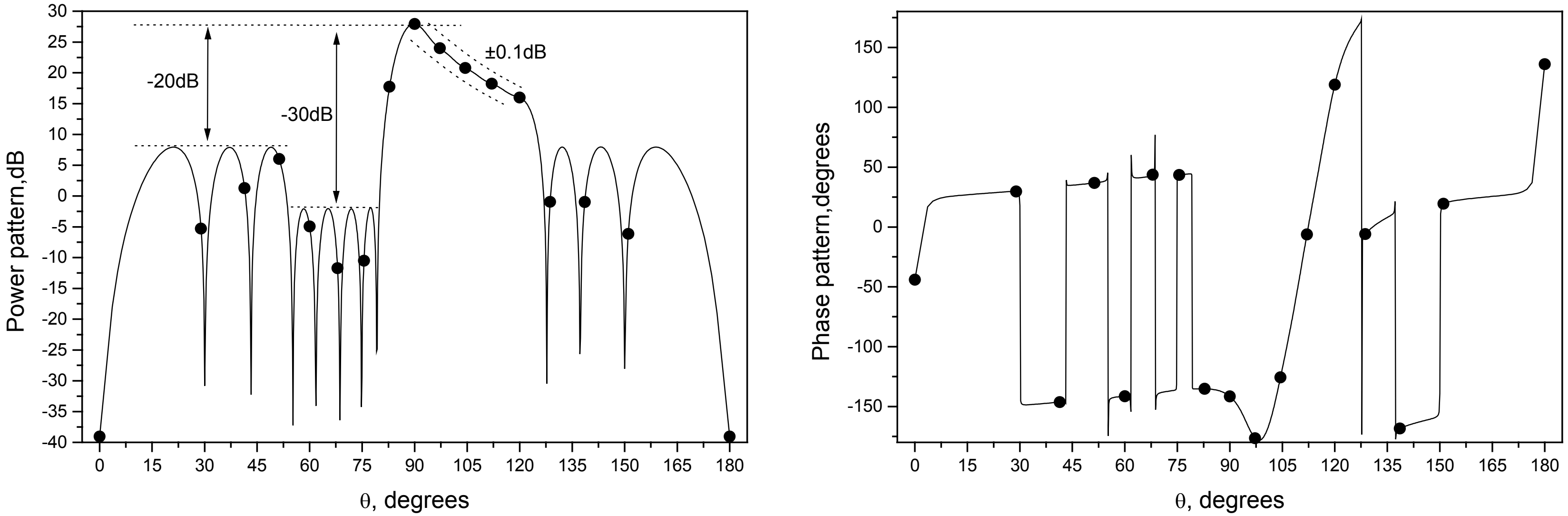



*Power pattern and phase pattern obtained (the samples are also shown)*

3 pairs of filled roots → $3^3$=27 different solutions that synthesizes the same normalized power pattern

## b) Complex pattern:

- Desired pattern: $\text{cosec}^2(\theta)\cos(\theta)$ pattern with a ripple requirement of ±0.1 dB, four −30 dB side lobes to the left of the cosecant beam, and all other sidelobes at -20 dB.
- This pattern is easily synthesized by using the original Orchard-Elliott method.




*Power pattern and phase pattern obtained (the samples are also shown)*

4 filled roots → $2^4=16$ different solutions that synthesizes the same normalized power pattern.

# CONCLUSIONS

- The proposed method obtains the Woodward-Lawson beam coefficients by sampling a previously synthesized pattern. Although this pattern could be obtained using any optimal pattern synthesis technique available in the previous literature, we have chosen the Orchard-Elliott method, which is computer-efficient, inexpensive and rapidly convergent.

- The combination of both synthesis methods will provide the advantages of each: on the one hand, power patterns with a controlled ripple and sidelobe topography and also a set of coefficients of the orthogonal beams which can be realized in real life with an array feed consisting of a Butler matrix.

- This method may be extended to direct broadcast satellite phased arrays by combining it with the sampling function method developed by Richie and Kritikos. In this case, the pattern to be sampled can be synthesized by using a recent quasi-analytical method that shapes the desired footprint as a composition of several $\phi$-symmetric circular Taylor patterns exhibiting flat-topped beams.